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Three-dimensional reconstruction of curves from pairs of projection views in the presence of error. II. Analysis of error

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ABSTRACT

We have previously described an approach to 3D intracerebral vascular reconstruction that uses an MRA as a reconstruction base. Additional vessels seen only by angiography are added by segmenting 2D curves from projection angiograms and reconstructing these curves into 3D, building upon the MRA.

This paper is the second of two that discuss the specific problem of reconstructing a 3D curve from a given pair of 2D curves in the presence of error. The method presented is capable of detecting and handling many errors produced by misregistration, image distortion, or misdefinition of 2D curves. The first paper gives an algorithm. The current paper discusses factors affecting the accuracy of a reconstructed curve, with emphasis upon registration error.

We analyze the spatial accuracy of a reconstructed point in terms of the relationships between pixel size, relative viewing angle, 3D point location, and registration error. We provide a theoretical framework that, given the known error properties of a registration algorithm, allows optimization of the viewing geometry so as to produce the highest precision of point reconstruction. A major focus is the effect of registration error upon the reconstruction of a curve. We subdivide registration error into two types, one of which produces smoothly continuous point placement errors and the other of which produces pixel pairing errors. We test our ability to reconstruct a 3D curve in the presence of both. Finally, we summarize approaches to other sources of error. We conclude with a list of recommendations to optimize reconstruction accuracy. When projection points are associated by the rules of epipolar geometry, viewplane point displacements should not exceed 1.5 - 2mm along the axis perpendicular to epipolar planes.

KEY WORDS: angiography, intracerebral vessels, few-view reconstruction, 3D curve, vascular reconstruction

I. INTRODUCTION

We have described a theoretical method of creating detailed, 3D maps of the intracerebral circulation from projection angiographic images (angiograms) and a magnetic resonance angiogram (MRA) obtained from the same patient[1,2]. The MRA is used as a reconstruction base. Additional angiographic information not present in the MRA is reconstructed into 3D, building upon the MRA. This process requires five steps: correction of image distortions, segmentation of MRA and angiograms[3, 4], registration of each angiogram with the MRA[5], association of 2D curves on projection views[2], and reconstruction of 2D curve pairs into 3D.

This paper is the second of two that discuss reconstruction of a 3D curve from a given pair of 2D curves in the presence of error. The companion paper provides methods[6]. The current paper analyzes errors that may complicate reconstruction, with emphasis upon misregistration. Methods of automatically associating 2D curves are discussed elsewhere[2], but depend upon the curve creation techniques described here.

II. THEORY

This section is divided into 4 portions. The first two provide definitions and an outline of our registration method. We then analyze factors affecting the precision of 3D point reconstruction. Finally, we discuss the effect of registration error on reconstruction of a 3D curve.

A. Definitions

We use the terms *2D curve* and *3D curve* to refer to the skeletons of 2- and 3D tubular objects. The reconstruction of a 3D curve involves selective pairing of points along a pair of 2D curves. The diameter of each reconstructed 3D tube is calculated by the viewing geometry, the location of each 3D point, and the width of the 2D tubular objects used to create the 3D tube[6].

When 2D curves are described in pixels, each ray becomes a pyramidal volume with its tip at the ray source. A *pixel intersection* is an intersection between a pair of pyramidal rays. An *epipolar volume* is the generalization of an epipolar plane when pixels are used. The course of a 3D object within an epipolar volume cannot be determined from a pair of projection views[6].

A *viewplane error* is any error in the set of points used to define a 2D curve. When viewplane errors interrupt reconstruction, the program searches along each 2D curve for nearby pixels capable of a pixel intersection. This approach "holds" the two curves together[6].

Reconstruction accuracy refers to the accuracy of a reconstruction as compared to the 3D coordinates of the vessels used to create projection views. Three factors contribute to reconstruction accuracy: the precision of 3D point placement, the accuracy of pixel-pairing along a given pair of 2D curves, and the accuracy of 2D curve association. This paper discusses the first two factors. Tests of 2D curve association are discussed elsewhere[2].

B. Registration method, coordinate systems, and definition of registration error

The pose of the angiographic camera is determined by individually registering each angiogram with an MRA. The 3D/2D registration process uses as primitives 6-20 2D curves extracted from the angiogram at sub-pixel resolution and an equivalent number of 3D curves extracted from the MRA at sub-voxel resolution. An approximation of the initial camera pose is used to initialize the algorithm. Association between 2 and 3D curves is established manually. The program then optimizes a viewplane based disparity measure based on the iterative closest point paradigm[7] between the angiogram skeletons and the projections of the MRA skeletons. Newton's method on the pose parameters in 3D is used to iteratively refine the solution. Iterations are halted when convergence criteria are met or after a preset number of iterations[5].

This method registers one projection view at a time with an MRA. The term *registration error* therefore always refers to miscalculation of the pose of the MRA relative to the pose of a SINGLE ray source and associated viewplane.

This paper uses 2 coordinate systems to explain different concepts. Section II.C.2 and Figures 1-2 employ the *patient coordinate system*. The MRA is viewed as fixed in space. During registration, each ray source and associated viewplane are translated and rotated relative to the MRA. This coordinate system demonstrates most clearly the effect of registration error upon the intersection of a pair of rays within a volume.

The majority of this paper uses the *viewplane coordinate system.* Each ray source is viewed as fixed at the origin. The viewplane, also fixed, is pierced centrally and orthogonally by the z axis. The y axis is vertical. During registration, the MRA is translated and rotated by the inverse of the geometrical transformation matrix used by the patient coordinate system. A "registration error of 1mm in z" therefore means that the position of a 3D point is miscalculated by 1mm along the axis connecting the ray souce and viewplane. This coordinate system best illustrates the sensitivity of reconstruction to registration error along different axes.

C. Factors affecting the accuracy of a reconstructed point

1. Overview

When 2D curves are defined in pixels, each ray becomes a pyramid. Each reconstructed point is then formed by intersection of a pair of pyramids. There are 4 consequences to this observation.

First, pixel-based reconstruction is fuzzy. The larger the pixel, the larger the spatial pyramid associated with each pixel, the larger the intersection volume of each pair of pyramidal rays, and the greater the uncertainty of each 3D point's location.

Second, the shape of a pyramidal intersection is related to the relative angles between the two rays. Narrowly separated rays produce elongated intersections, with resultant uncertainty of the depth of the reconstructed point.

The remaining 2 consequences affect reconstruction in the presence of error. In each case, there is a trade-off betweeen precision of 3D point placement and resistance to viewplane error.

Viewplane errors may displace 3D point projections so that a pair of rays that should intersect cannot. The correct 3D point then cannot be created. With large pixels, an erroneously placed 2D point is more likely to remain within the correct pixel, thus permitting intersection of associated rays. Large pixels therefore decrease the chance of pixel-pairing errors, at the cost of decreasing the precision of 3D point placement.

Similarly, some regions of space are more sensitive to registration error than others. A small shift in position of a 3D point close to the ray source may produce a large shift in its projection. However, 3D points close to the ray source are contained within the tips of spatial pyramids, so the volume surrounding each point is small. Reconstruction from highly magnified views will therefore provide better spatial resolution but will also be more sensitive to registration error.

2. Error analysis

This section provides a quantitative analysis of many variables affecting the precision of 3D point reconstruction. Given the known error properties of a registration algorithm, one may use the model below to determine the relative advantages of various viewing geometries.

In order to best analyze ray intersections, this section employs the patient coordinate system described earlier. It is assumed that paired point projections are known. The *optical axis* is the ray between ray source and associated viewplane that pierces the viewplane orthogonally.

In the absence of error, a 3D point and its two viewplane projection points define a pair of intersecting line segments, each originating at a ray source and terminating at a projection point. Viewplane error may displace a projection point on the viewplane. The uncertainty of each projection point's location can be modeled as a circle of radius δ, centered on the true projection point. By this model, each ray becomes a cone with its apex at a ray source. The region of uncertainty of each reconstructed point then becomes the intersection volume of two cones.

During registration, the patient coordinate system views the MRA as fixed while each ray source and associated viewplane, separated by a set distance, "move" relative to the MRA. A 3D/2D registration error therefore produces an error in the location of a projection point (a viewplane error), an error in the calculated position of the ray source (a *ray source error*), or both. Viewplane uncertainty can be modeled as a circle on the viewplane, as discussed above. Ray source uncertainty may be visualized as a volume. The apex of each "ray" lies within this volume.

Figure 1 illustrates the region of uncertainty of a reconstructed point P as defined by the intersection of a pair of truncated cones. Subscripts l and r respectively denote the left and right viewplanes. E_l and E_r are the two ray sources and P_l and P_r are P's two projections. θ is the reconstruction angle between projection rays. ψ is the angle between the projection ray and the viewplane. Finally, δ is the uncertainty in locating a projection point on the viewplane.

The volume of intersection of two cones can be approximated by a parallelepiped defined by the vectors $\vec{e}_{up}$, $\vec{e}_l$ and $\vec{e}_r$. The directions of $\vec{e}_l$ and $\vec{e}_r$ are respectively parallel to the lines PE_r and PE_l. The direction of $\vec{e}_{up}$ is normal to the epipolar plane defined by P, E_l, and E_r. A straightforward geometrical solution[8] can be used to determine the magnitudes of $\vec{e}_l$, $\vec{e}_r$, and $\vec{e}_{up}$. In the following equations, τ represents the ratio of source to target and source to viewplane distances, and Δz and Δr represent respectively the magnitude of the uncertainty of the ray source location along the optical axis and in the plane perpendicular to the optical axis.

$$|\vec{e}_l| = \frac{1}{\sin\theta}\left[(1-\tau_l)(\Delta z_l \cos\psi_l + \Delta r_l \sin\psi_l) + \tau_l \delta_l \sin\psi_l\right] \qquad \text{eq. 1}$$

$$|\vec{e}_r| = \frac{1}{\sin\theta}\left[(1-\tau_r)(\Delta z_r \cos\psi_r + \Delta r_r \sin\psi_r) + \tau_r \delta_r \sin\psi_r\right] \qquad \text{eq. 2}$$

$$|\vec{e}_{up}| = \sin\theta \ (\max(\vec{e}_l, \vec{e}_r)), \qquad \text{eq. 3}$$

where $\tau_i = \dfrac{E_i P}{E_i P_i}$ for $i \in \{l, r\}$

A simple upper bound on the error of a reconstructed point is given by $[\,|\vec{e}_l|^2 + |\vec{e}_r|^2 + |\vec{e}_{up}|^2\,]^{1/2}$. In the following discussion, variables without subscripts apply equally to equations 1 and 2.

The equations have an intuitive interpretation. The precision of a reconstructed 3D point is affected by several sources of error which manifest themselves either as viewplane error or as ray source error. The term $\delta \sin\psi$ is the contribution of viewplane error. The term $(\Delta z \cos\psi + \Delta r \sin\psi)$ is the contribution of ray source error. The relative contribution of each to the final error is determined by the ratio of source to target vs. source to film distance (τ).

This error model facilitates understanding of the various factors contributing to the precision of 3D point reconstruction. Reconstruction angle is the single most important factor since θ has a multiplicative effect on $\vec{e}_l$ and $\vec{e}_r$. When $\theta > 30^o$, $\frac{1}{\sin\theta}$ is relatively small (less than 2). However, when $\theta < 30^o$ the value of $\frac{1}{\sin\theta}$ rises sharply. Reconstruction from narrow angle views therefore constitutes a problem that becomes close to ill-conditioned.

τ permits reconstruction precision to be biased toward errors either on the viewplane or at the ray source. Different registration algorithms will produce different viewplane and ray source errors. Our method converges to a solution in which viewplane errors are much smaller than ray source error (errors in the range of 10^{-4} mm on the viewplane and 1mm at the ray source in the absence of image distortion)[5, 8]. For us, the effects of registration error upon reconstruction are therefore reduced by placing the patient's head close to the viewplane.

In the same fashion, ψ allows one to trade off ray source errors parallel or perpendicular to the optical axis. Our registration algorithm locates the ray source more accurately in the plane perpendicular to the optical axis than along it. As ψ approaches 90 degrees, the effect of Δz on reconstruction precision vanishes. For our registration method, the effect of ray source error upon reconstruction is therefore reduced when the region of interest is centered on the viewplane. Different registration methods can use this general model to determine their own optimal viewing geometries and to evaluate the effects of different viewing geometries.

D. Registration error and 3D curve reconstruction

The previous analyses have assumed that the paired projections of a 3D point are known. Reconstructing a 3D curve in the presence of error is more difficult since registration error (and other viewplane errors) may confound correct pairing of 2D points.

We first define the terms P_{3D}, *P1*, *P2*, *P2’, EP*, *E1*, and *E1’*. When both projection views are perfectly registered with a volume, each 3D point P_{3D} is associated with the projection points *P1* and *P2*. These 3 points define an epipolar plane *EP*. *E1* is the epipolar line on the second viewplane. *P1* can only pair with a point crossed by *E1*. If P_{3D} is misregistered relative to the second viewplane, however, both its new projection *P2’* and epipolar line *E1’* may be displaced.

It is useful to divide registration errors into two types: those that misplace P_{3D} within *EP* and those that misplace P_{3D} outside it. The first error type miscalculates the location of ray intersections. The second type produces unpredictable pixel-pairing errors (Fig. 2).

Consider registration error applied to the second viewplane. For the first error type (P_{3D} is displaced within *EP*), *E1’* will still cross *P2*. The rays defined by *P1* and *P2* still intersect, although the location of the 3D point is incorrect (Fig 2). The severity of the 3D point placement error depends on the angles between intersecting rays and on the distance between *P2* and *P2’*.

The second, more serious error type translates P_{3D} out of *EP*. If such misregistration is applied to the second view, the epipolar line is misdrawn and no ray intersection can occur between *P1* and *P2* (Fig 2). The effect upon reconstruction is then dependent upon the configuration of the 2D curves. If the second viewplane’s 2D curve is directed along a vector non-parallel to *E1*, *P1* will miss *P2* but may map successfully (and incorrectly) to a nearby point on the 2D curve. However, if the second 2D curve is parallel to the epipolar line, no points will be mapped (Fig 3). The reconstructed curve will thus be shifted by an amount which varies with the slope of 2D curves relative to epipolar lines.

This second type of registration error has multiple effects. Reconstructed curves lose their tips. The number of reconstructed points decreases. Tops and bottoms of loops are cut off.

Finally, pixel-pairing errors create 3D points located far from the true 3D curve. For example, for 2D curves with sine wave configurations, a trough point on one curve may only combine with a peak point on the other (Fig 3c). *P1* thus pairs with a pixel displaced both in x and y, creating a badly displaced 3D point. Other 2D curve configurations produce similar errors (Fig 3d).

As stated earlier, narrowly separated rays magnify 3D point placement errors. Narrow angle reconstructions therefore increase the effect of either type of registration error.

III. METHODS

A. Reconstruction of a point

We examined the effects of viewing angle and pixel size on the precision of 3D point reconstruction by projecting 3D points upon two viewplanes rotated by 2-90 degrees relative to each other. The source to viewplane distance (914 mm), viewplane size (292 mm), and magnification factor (1.8) were those used during angiography of the patient whose MRA was employed.

Each projection point was modeled as the center of a square pixel. Each pixel defined a pyramidal ray. Half of the maximum distance between vertices of the pyramid-pyramid intersection volume was used to define maximum point placement error.

Tests were performed with viewplanes subdivided into 256 x 256, 512 x 512, 1024 x 1024, and 10240 x 10240 pixels. For a viewplane of 1024 x 1024 pixels, pixel size was slightly less than 0.3 x 0.3 mm. Viewplanes subdivided into a larger or smaller number of pixels contained proportionately smaller or larger pixels.

B. Reconstruction of a 3D curve in the presence of registration error

1. Wide angle views

We tested the effect of registration error upon 3D curve reconstruction by projecting vessels extracted from MRA datasets, and then reconstructing 3D curves from pairs of 2D curves as described in the companion report[6]. The two views used for reconstruction simulated clinically used AP and lateral angiographic views subdivided into 1024x1024 pixels of 0.3 x 0.3 mm. The magnification factor was about 1.8.

Tests were performed on 3 sets of vessels segmented from 2 MRA datasets. Each test reconstructed 40-60 vessels and analyzed 1100-1500 3D points. Each test included 40 program runs to analyze the effect of progressive registration error applied to one viewplane at a time.

In theory, registration errors along different axes should affect reconstruction differently. Registration error was therefore applied independently in x, y, and z of a viewplane coordinate system by mistranslating the entire 3D dataset relative to one viewplane along one axis. Registration errors were examined in increments of 1mm from 1-5 mm in x and z, and in increments of 0.2 mm from 0.2-2 mm in y.

Reconstruction accuracy was assessed by comparing each reconstructed vessel with the associated segmented MRA vessel. Parameters analyzed included the mean and standard deviation of the distance between each reconstructed 3D point and the closest 3D point on the associated 3D curve, the maximum point placement error, the maximum gap between the calculated and true start and endpoints of each 3D curve, the number of vessels that failed to map, and a qualitative comparison of the reconstructed and true vessel's appearance as seen from 3 mutually orthogonal points of view.

Criteria for clinical utility for surgical planning included a reconstruction that appears correct or close to correct as seen from 3 mutually orthogonal angles of view, with mean point placement errors of 2 mm or less. For the 3D coordinate determinations required by stereotactic surgery, additional criteria included no gap or point placement error of more than 5 mm[2, 6].

2. Narrow angle views of varying pixel size

We performed limited tests using views separated by small relative angles, reconstructing one of the datasets above. The relative viewing angles employwed were 18, 25, 45, and 60 degrees. Results were analyzed for translational errors of 0.4-0.8 mm in x and y. Viewplanes were subdivided into 10240x10240 (0.03 mm) pixels for angles of 18-45 degrees, 1024x1024 (0.3 mm) pixels for angles of 45-60 degrees, and additionally into 512x512 (0.6 mm) pixels for an angle of 60 degrees.

IV. RESULTS

A. Reconstruction of a point

Figure 4 illustrates the effect of relative viewing angle on spatial resolution of a reconstructed point. Results are shown for a point close to the center of the patient's head, and agree with the theoretical analysis provided in section II.C.2. For any given pixel size, the the shape of the curve relating viewing angle to spatial resolution resembles a multiple of $1/\sin\theta$, with flattening of each curve and improvement in the precision of the reconstructed point for angles greater than about 30 degrees.

Spatial accuracy of point placement is thus poor for narrowly separated views. In order to double spatial precision, one must subdivide each pixel into 4. Pixel size could theoretically be adjusted to provide any desired resolution for a given magnification and viewing angle. However, as shown in section B.2 below, small pixels produce more pixel-pairing errors.

B. Reconstruction of a curve in the presence of registration error

1. Wide angle views

Tests were performed in 3 datasets, using AP and lateral views and viewplanes subdivided into 1024x1024 (0.3 mm) pixels. We use the term *offpoint* for a point displaced more than 5 mm from its curve and *start*- and *end-jump* to indicate the gap between the calculated and true start and endpoints of a 3D curve. The following discussion employs the viewplane coordinate system described earlier. Results are shown in Figure 5.

1. Registration error in x

For reconstruction performed from AP and lateral views, the major effect of mistranslating a 3D point in x in one viewplane's coordinate system is to displace the 3D point within the epipolar volume. The predominant effect on reconstruction is thus upon mean point placement (Fig 5).

The results obtained were similar for all 3 datasets. Mean point placement error followed almost linearly the magnitude of registration error. Mistranslation by 2 mm produced mean point placement errors of slightly less than 2 mm. Gaps of more than 5 mm and creation of offpoints first appeared with mistranslations of 3-4 mm. We conclude that, for the particular views used, reconstruction producing less than 2 mm mean point placement error requires registration accuracy within 2 mm in x. This distance on the viewplane was about 12 pixels or 3-4 mm.

2. Registration error in z

The z axis represents depth. Misregistering the patient's head in z changes the projections of all 3D points by a magnitude dependent upon the point's z coordinate. For the viewing parameters employed, a 5 mm translation in z exerted approximately the same effect on point projections as the sum of a 1 mm translation in x plus a 1 mm translation in y.

Z misregistration had relatively little effect on reconstruction accuracy in our test situation (Fig 5). Mean point placement errors were less than a millimeter for all tests. A single vessel developed an offpoint with misregistration of 4 mm. We conclude that excellent reconstruction can be performed with z registration errors of 4mm.

3. Registration error in y

For the viewing parameters employed, the primary effect of misregistration in y is to misdraw epipolar lines, leading to pixel pairing errors. Multiple problems occurred, including point placement errors, loss of the ends of reconstructed vessels, cut-offs of loops, decrease in the number of 3D points, and mapping failure of short, horizontal vessels. Point placement errors were most severe when curves had the configuration shown in Figure 3D. Such configurations were fortunately rare, but two datasets contained a combined total of 3 such vessels.

The pattern of failure differed for the 3 datasets (Fig. 5). A start- or end-jump of more than 5 mm first appeared with misregistration of 0.4-0.6 mm in 2 datasets. For the third dataset, in which vessel projections coursed more vertically, such jumps did not occur until misregistration of 1.2 mm. Point placement errors of more than 5 mm first appeared in all 3 datasets with y misregistration of 0.8-1.0 mm. With misregistration of 2 mm, so much data was lost that the results were useless. Mean point placement error tended to mirror the amount of registration error, indicating that despite gaps and the presence of occasional wildly placed points, reconstruction accuracy remained good for the hundreds of points mapped (Fig 5).

We conclude that adequate reconstruction from this pair of clinically used views requires registration accuracy in y of better than 1 mm. On the viewplane, this distance is 1.5-2 mm. Even greater registration accuracy is required to prevent gaps of 5 mm or more in reconstructed vessels if vessels possess strings of points within the epipolar volume. For stereotactic surgery, registration accuracy of 0.4 mm in y (point projection errors less than 0.8 mm) may be required.

2. Narrow angle views of varying pixel size

For narrow angle views, y misregistration exhibited even larger effects. All tests exhibited errors, even with y translational errors of only 0.4 mm. The best result used views separated by 60 degrees and a viewplane subdivided into 512x512 (0.6 mm) pixels, producing the marginally acceptable result of 1 offpoint and 1 start-jump error, each barely above 5 mm.

In two cases (reconstruction from relative angles of 45 and 60 degrees), we tested the effect of pixel size upon reconstruction accuracy. Larger pixels provide poorer spatial precision but better prevent pixel mispairings. In both cases, pixel size had little effect upon maximum point placement error. However, the larger pixel size prevented some mapping failures and reduced the number of vessels exhibiting large start- or end-jumps.

We conclude that accurate reconstruction will require views separated by at least 60 degrees. Large pixels confer partial protection from the effects of registration error.

V. DISCUSSION

A. Errors in reconstruction of points and curves

This paper discusses the precision of 3D point placement (or inability to create a point) as the result of relationships between pixel size, ray angles, viewplane error, and point location. Henri discusses the relationship of pixel size and viewing angle to “minimum resolvable depth”, concluding that little improvement is gained by increasing viewing angle beyond 15 degrees[9]. Although we do not disagree with Henri’s analysis, our concomitant examination of additional variables leads to the more conservative conclusion that clinically useful reconstruction will require views separated by at least 60 degrees.

Registration error is a focus of this report. We expand upon Henri's observation that registration error may cut off the ends of 3D curves[10] and analyze the different effects of registration error along different axes. We test reconstructions of realistically complex input data and conclude that, for a particular pair of clinically used views, registration accuracy is required so as to displace point projections on the viewplane by no more than 3-4 mm in x and 1.5-2 mm in y. For stereotactic surgery, twice the accuracy in y may be required. Four points should be made about these specific conclusions, however.

First, these specific results apply to a particular imaging geometry. For more narrowly separated views, registration (and 2D curve definition) must be more accurate. Our results also indicate selective sensitivity to misregistration along the y axis. The axis of greatest sensitivity to error is the vector perpendicular to epipolar planes in the area of interest. A pair of differently oriented views may define differently oriented epipolar planes and thus exhibit greatest sensitivity to error along a different vector.

Second, the effect upon reconstruction of a given 3D/2D registration error is dependent upon magnification. If a 3D point is close to the ray source, a small shift in that 3D point's position may produce a large shift in its projection point. Our test reconstructions were performed from simulated angiograms with a magnification of about 1.8. If more highly magnified views had been used with the same registration error the results would have been worse; if less magnified views had been used the results would have been better. In general, reconstruction of a 3D point close to the ray source demands higher registration accuracy than does reconstruction of a 3D point close to the viewplane. Section II.C.2 provides means of estimating the effect of different viewing geometries upon the precision of 3D point reconstruction when the error properties of the 3D/2D registration algorithm are known.

Third, reconstruction accuracy is partially dependent upon curve configuration. 3D curves are best reconstructed when they course perpendicularly to epipolar planes. Patient positioning during angiography could therefore affect results. The carotid circulation courses predominantly supero-posteriorly. For reconstruction from AP and lateral views, flexion of the patient's head during angiography might therefore improve results. Our reconstruction results support this hypothesis; the dataset least sensitive to reconstruction error contained the largest number of vessels coursing most perpendicularly to epipolar planes.

Finally, small pixels provide higher spatial resolution but are more likely to produce pixel-pairing errors. Our tests of narrow angle reconstructions suggest that, in the presence of error, the largest pixel size should be used that is compatible with "adequate" spatial resolution.

B. Implications of the proposed methods

This report describes reconstructions from viewplanes subdivided into matrices of 1024 x 1024 (0.3 mm) pixels. The resultant 3D detail is higher than that achievable by MRA. Although we have described tree-based display of vessels segmented from MRA[11], the information provided by segmented data is limited by voxel size. Reconstruction of 3D vessels from finely detailed projection images should provide significant additional information.

A sobering conclusion is that reconstruction of complex 3D curves from small pixels requires exquisite registration accuracy. This sensitivity to registration error is not unique to our reconstruction approach, but will apply to any method using epipolar geometry to associate projection points. 2D curves coursing perpendicular to epipolar lines are relatively insensitive to the effects of registration error, but complex, looping 2D curves are not.

Sufficient registration accuracy is difficult to achieve by traditional registration methods. Our method employs hundreds of points at sub-voxel and sub-pixel resolution[5], and is thus potentially more accurate than alternative methods based upon a small number of points. Tests of registration accuracy using undistorted, synthetic angiograms indicate that, following registration, the MAXIMUM error in locating a 3D point within the volume of interest is less than 0.1, 0.1, and 1 mm in x, y, and z of the viewplane coordinate system[5]. We have used this registration method to provide locally correct registration of MRA and true angiographic data (neither corrected for image distortion) in order to produce limited but clinically useful angiographic reconstructions[11]. Correction of image distortions is necessary before we can obtain sufficiently accurate global registration for large volume reconstruction, however.

C. Additional sources of error

Four additional sources of error must be addressed. The first is image distortion, which can exert disastrous effects upon reconstruction. However, Peuchot corrects distortions produced by the image intensifier to sub-pixel levels[12], and Michiels corrects magnetic and flow distortions in MRA data to sub-voxel levels[13, 14]. We therefore view correction of image distortions as essential to intracerebral vascular reconstruction but, since the problem has already been effectively addressed by others, do not discuss it further here.

A second source of error is misestimation of the source to film distance and of the location of the *principal point* (the orthographic projection of the ray souce on the viewplane). Liu has analyzed the effects of these errors upon registration accuracy in our own imaging system[8]. We agree with Fencil and Metz that, for clinically used views, misestimates of the source to film distance are well tolerated until such errors reach several centimeters[15]. However, we find that miscalculations of the principal point can produce errors when associated point projections are not known. Calibration methods such as those described by Willson[16] should be be used to determine the principal point when 2D point pairing is done by the rules of epipolar geometry.

A third potential source of error is erroneous 2D curve definition. However, we have successfully reconstructed a distal middle cerebral tree from two projections of an MRA, using only manual tracing to define points along 2D curves and linear interpolation to connect separated 2D points[1]. In the absence of image distortion we therefore view errors of 2D curve definition as less significant than the effects of registration error emphasized in this report.

Finally, a major source of error is faulty association of 2D curves. We have described and tested a method of automatically associating 2D curves in the presence of viewplane error[2]. This method depends upon the 3D curve reconstruction techniques described in the companion paper[6] and tested here. It is therefore not surprising that this method of 2D curve association begins to fail at levels of registration error similar to those at which reconstruction of individual 3D curves encounter difficulty, as described in the current report.

VI. CONCLUSIONS

This report studies sources of error that affect reconstruction of a 3D curve, with emphasis upon registration error. We provide a theoretical framework that, given the known error properties of any registration algorithm, allows optimization of the viewing geometry so as to produce the highest precision of point reconstruction. We include both theoretical and experimental evaluation of the tolerance of 3D curve reconstruction to error. Our results indicate that reconstruction by the rules of epipolar geometry is likely to require registration accuracy (and accuracy of 2D curve definition) such that points on the viewplane are displaced by no more than 1.5- 2mm along the axis perpendicular to epipolar planes. Even greater accuracy, producing maximum viewplane point displacements of 0.8mm, may be required to produce reconstructions useful for stereotactic surgery. If reconstruction is performed from views of small relative angles, even greater accuracy is required. This sensitivity to registration error is not unique to our particular approach, but will apply to any method using epipolar geometry to associate projection points.

As a result of these analyses, we offer the following suggestions.

1) For a pair of views rotated relative to each other around a single axis, the axis of rotation defines the vector in which reconstruction will be most sensitive to error. One should select a registration algorithm capable of providing high accuracy along this vector.

2) Views should be obtained in which 2D curves course predominantly perpendicularly to epipolar planes.

3) Views should be separated by at least 60 degrees.

4) Viewplanes should be subdivided into the largest pixels allowing “adequate” spatial resolution for any given magnification and viewing angle.

5) Magnification should be used so as to bias registration errors toward viewplane error or ray source error, dependent upon the registration algorithm used.

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FIGURE LEGENDS

Figure 1: Analysis of error in the epipolar volume

Figure 2. Two types of registration error. P1 and P2 are projections of a 3D point. E1 is the epipolar line on the second view. With misregistration of view 2, both the projection of the 3D point and E1 may shift position. **A.** Registration error type 1. E1 still crosses P2, and P1 and P2 can be paired. **B.** Registration error type 2. E1 does not cross P2, and P1 and P2 cannot be paired.

Figure 3. Effects on reconstruction of registration error type 2. The column on the left shows two perfectly registered projections of the same object. The column on the right shows misregistered projections. In A, the object courses perpendicularly to the epipolar plane. The top and bottom of the reconstructed object will be lost, but the intervening points can be reconstructed. In B, the object courses within the epipolar plane. It will fail to map entirely. In C, misregistration applied to a sine shaped curve makes possible the reconstruction of only a single point. The pixel-pairing error will produce a 3D point far from the true curve. D shows the effect of misregistration upon a curve pair that each possess two components approximately perpendicular to the epipolar plane with a long intervening segment within the epipolar plane. Reconstruction in the presence of misregistration will produce a 3D curve that misses its start and end segments but is otherwise correctly mapped throughout its vertical course; a single, severely misplaced 3D point will be reconstructed when the top of the first vertical segment on view 1 maps to the bottom of the second vertical segment on view 2.

Figure 4. Effects of pixel size and relative viewing angles on spatial resolution of a reconstructed point. Results are shown for viewplanes subdivided into 512x512 (0.6 mm; ♦), 1024x1024 (0.3 mm; []), and 10240x10240 (0.03 mm; Δ) pixels.

Figure 5. Effect on reconstruction accuracy of registration error applied to the lateral view. Each graph depicts results from 3 datasets, represented respectively as diamonds, squares and triangles. The abscissa of each graph shows translational error of the 3D dataset along one axis. The first 6 graphs give the mean magnitude of error and maximum point placement errors in mm under conditions of x,y, and z misregistration. Note that mean point placement errors increase linearly with increasing translational errors along all 3 axes but that maximum point placement errors increase rapidly with only small amounts of misregistration in Y. Y misregistration also produces significant loss of data. The maximum jump distance in mm between a vessel's calculated and true startpoint or calculated and true endpoint is shown in graph 7. This distance is indicative of the gaps produced in reconstructed vessels. The average number of points per vessel also drops with increasing errors in Y misregistration, as shown in graph 8.

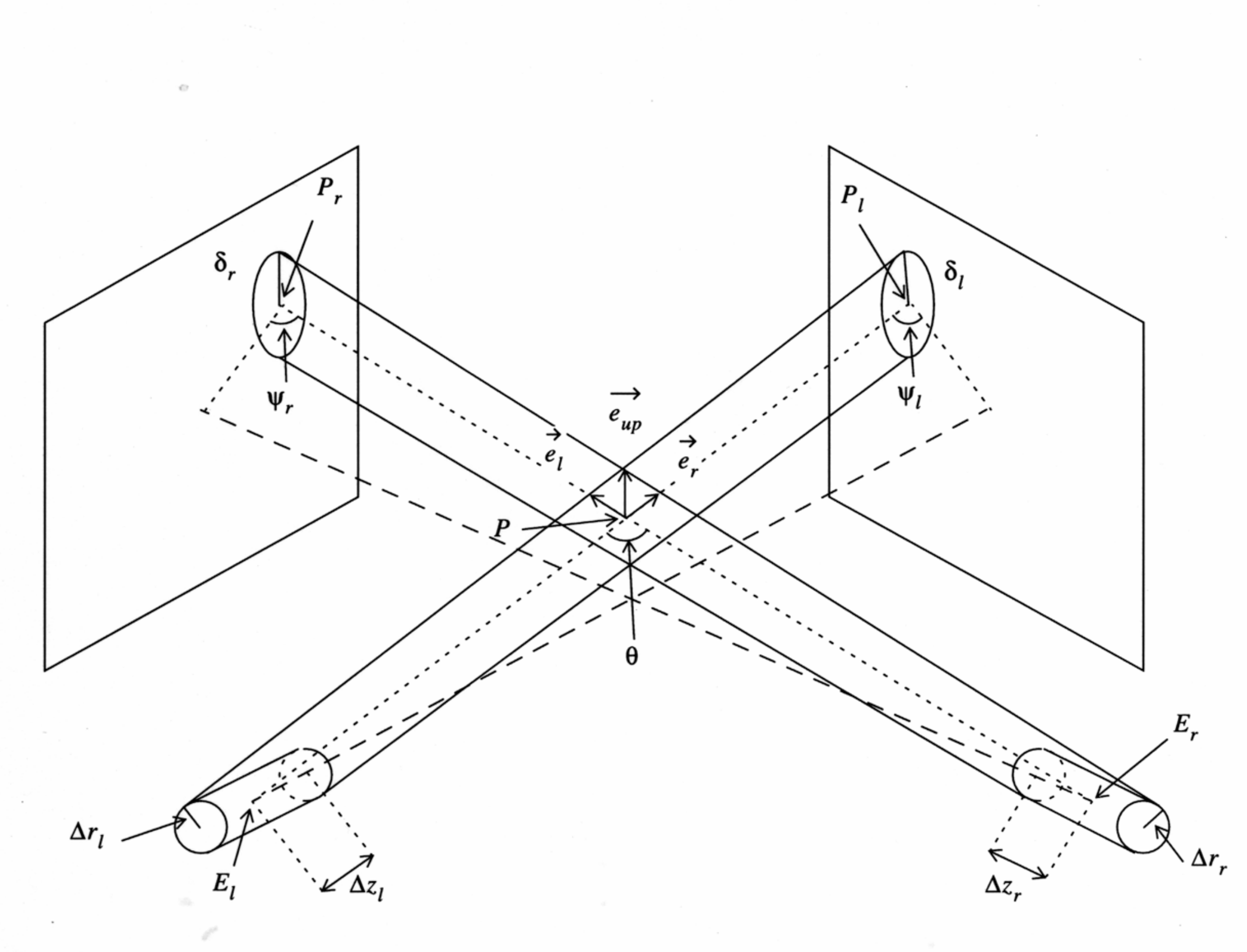
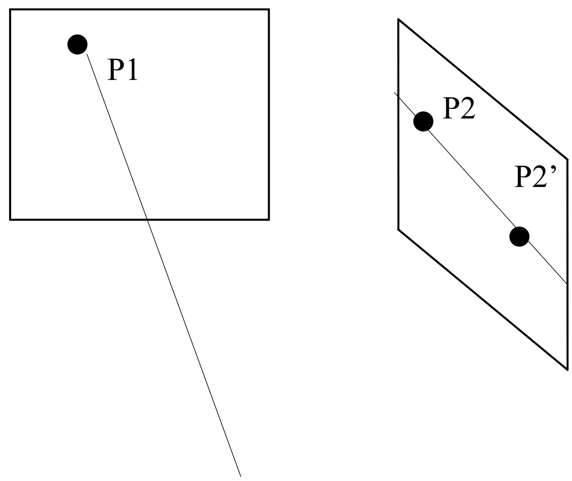
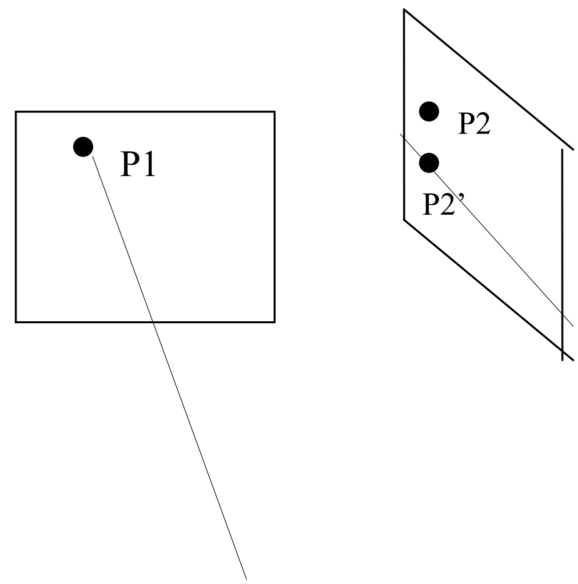


Fig 1. Bullitt et al. (Error analysis) MPH



(a)



(b)

Fig 2. Bullitt et al (Error analysis) MPH

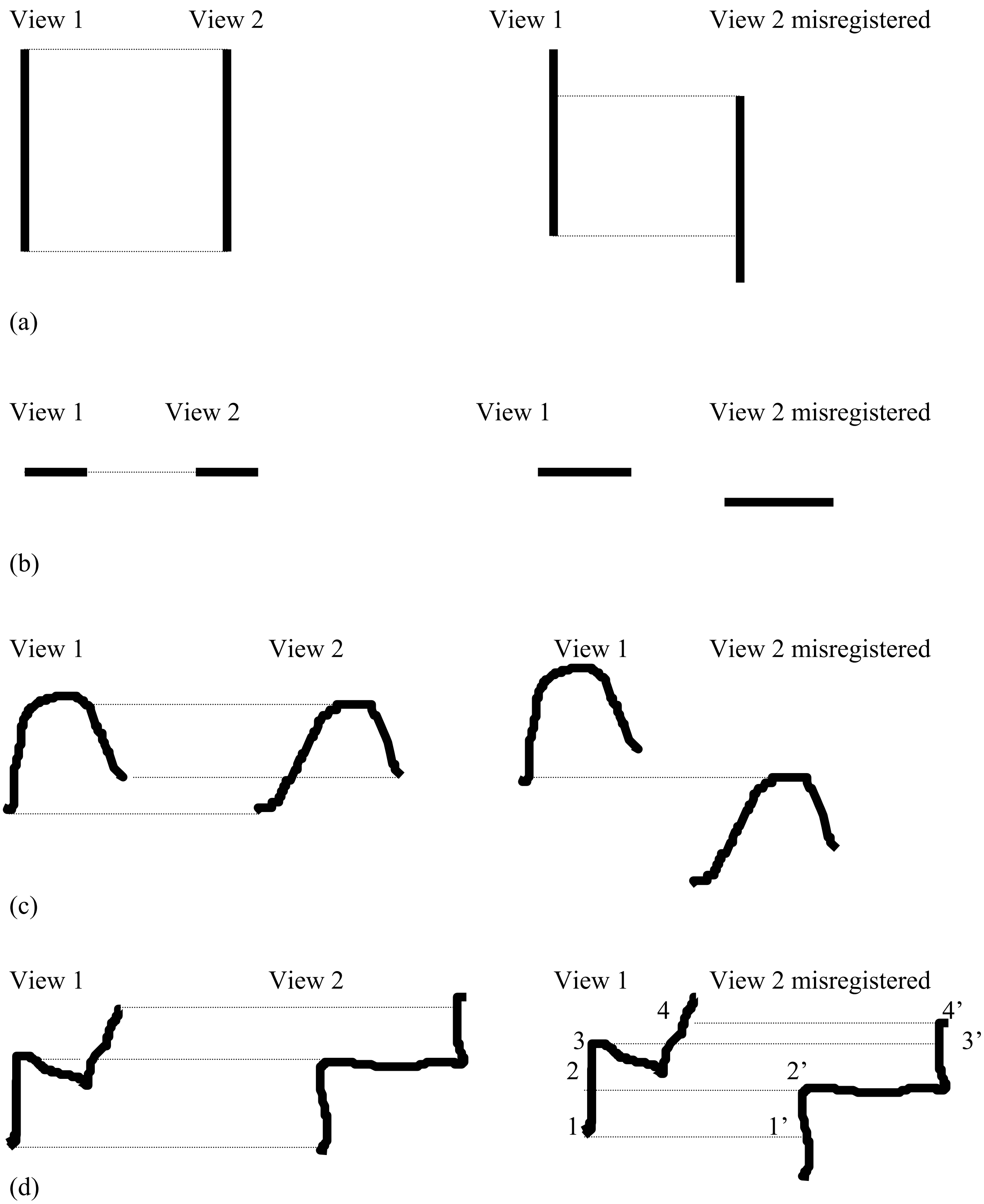


Fig 3. Bullitt et al (Error analysis) MPH

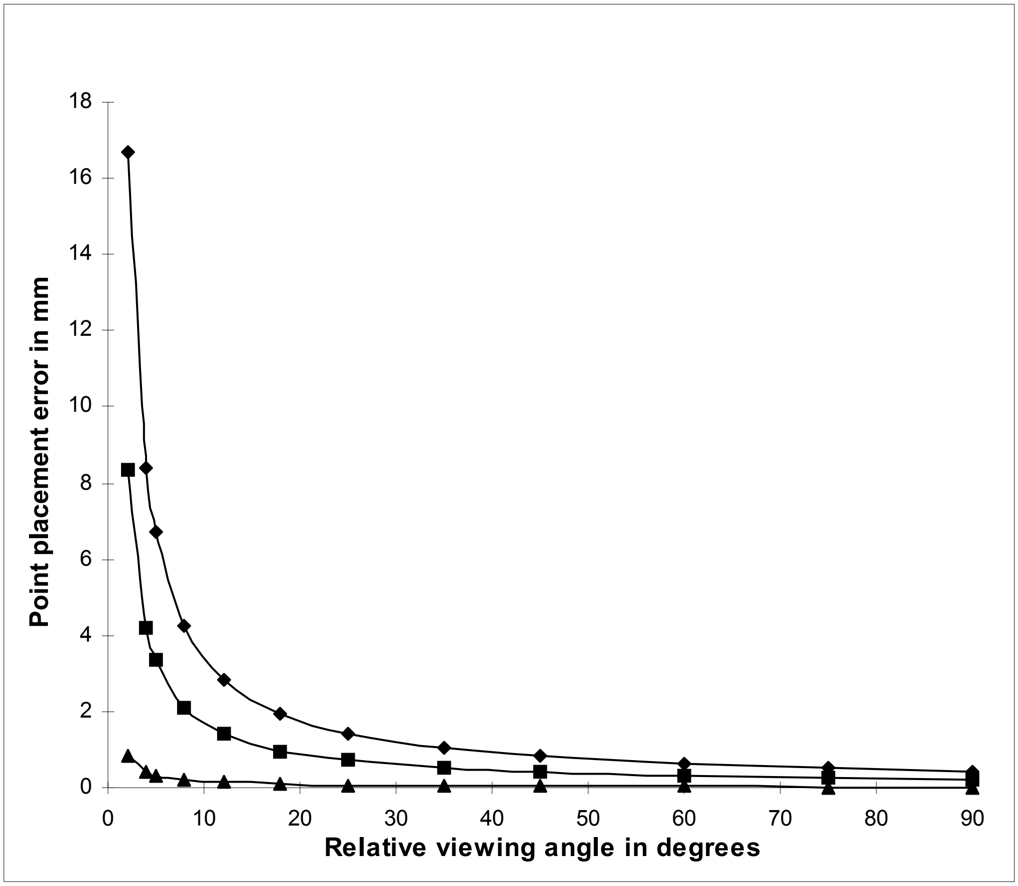


Fig 4. Bullitt et al. (Error analysis) MPH

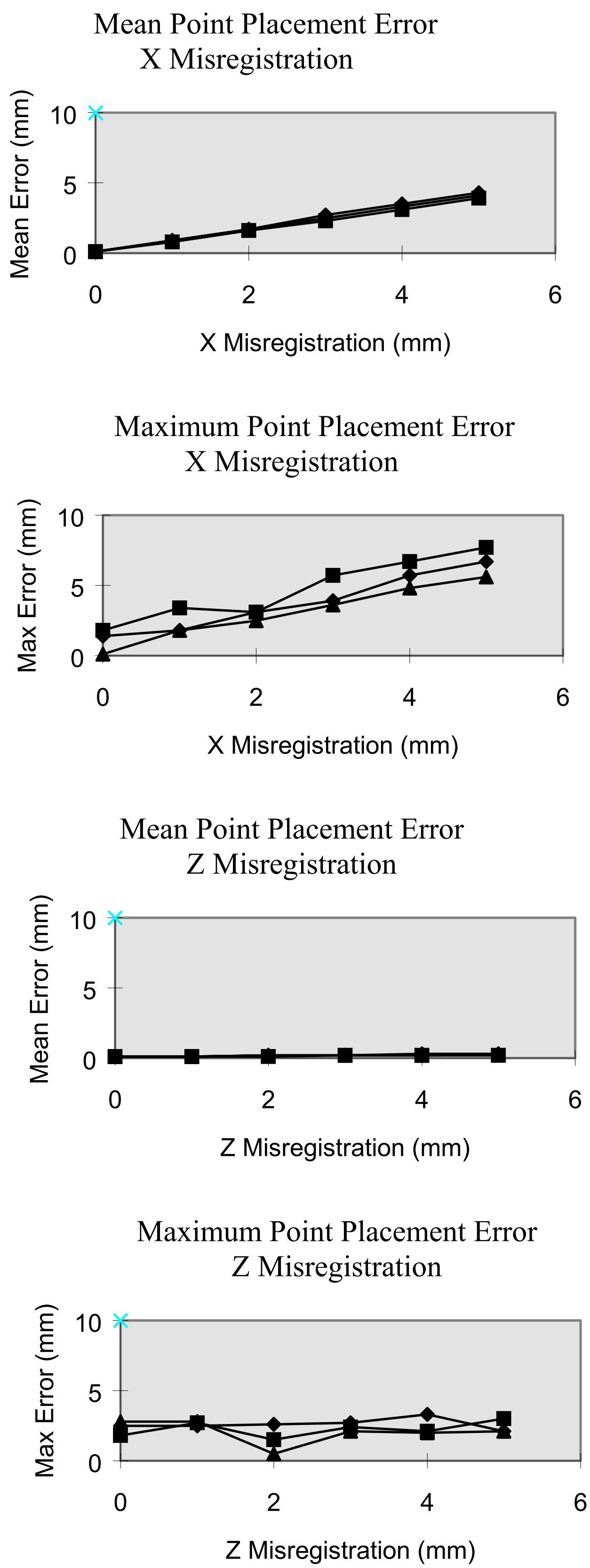


Fig 5 (page 1 of 2). Bullitt et al. (Error analysis) MPH

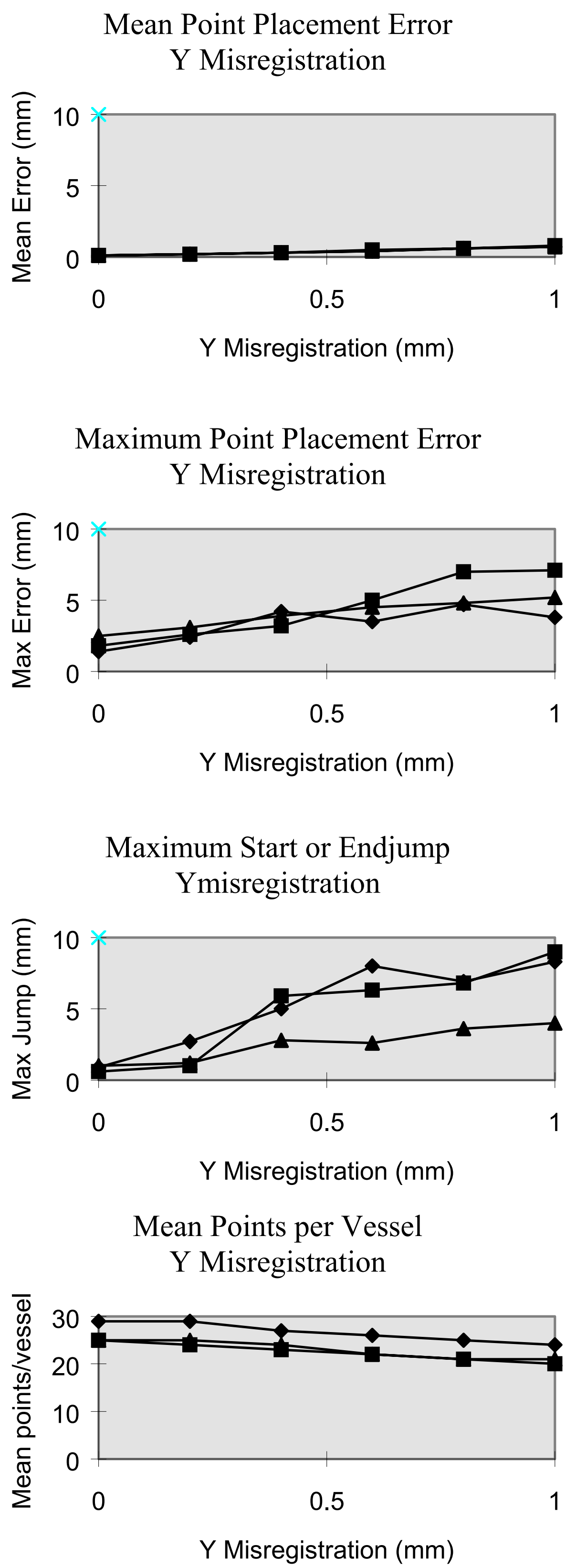


Fig 5 . Bullitt et al. (Error analysis) MPH